A UNIFIED THEORY FOR THIN WIRE ANTENNAS OF ARBITRARY LENGTH[†]

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I. Introduction

Currently, several different theories are necessary to describe the currents on a cylindrical transmitting antenna for different ranges of the electrical length, 2kh, and electrical radius, ka. Thin wire theories such as Shen, Wu, and King's [1] are appropriate only for thin ($ka \ll 1$) antennas whose electrical length is somewhat greater than $kh = 1$. The iterative technique of King and Middleton [2, Sec. II.30] depends primarily upon the parameter, $\Omega = 2\ \ell n(2h/a)$, being large while at the same time kh may not be too small nor ka too large. King's [2, Sec. II.31] short antenna theory is applicable to cylindrical antennas having a large Ω and electrical lengths, $kh < 1$, but involves graphical interpolations and is awkward to use in general applications. Wholly numerical approaches, such as the moment method of Harrington [3] do not have any practical limitations on either kh or ka, although computation times become excessive if the antenna is either very long or short, or very thick. This piecemeal approach to the analysis of the cylindrical transmitting antennas presents significant difficulties in problems involving large bandwidths such as time transient investigations. Our reexamination of the Wiener-Hopf technique as applied to cylindrical antenna problems reveals that simple asymptotic solutions for the current distribution are dependent upon essentially the same condition as used by King and Middleton [2, Sec. II.30]. And with a few simple modifications, our theory may be extended to include even quite thick ($ka \approx 1$) cylinders. Hence, for an antenna whose length is much greater than its radius, i.e., $\Omega = 2\ \ell n(2h/a)$ is large, our theory is valid over a very wide frequency range, in which the electrical length passes from short to very long at the same time the electrical radius passes from very thin to quite thick. In addition, this theory is shown to give accurate input conductance values for thick antennas where Ω is not very large.

II. Finite Length Cylindrical Transmitting Antenna

Under the condition, which is quite similar to that used in King and Middleton's [2, Sec. II.30] analysis, that the parameter,

$$\Omega = 2\ \ell n(2h/a) \gg |\ell n(2kh)|, \qquad (1)$$

the total z-directed current on a finite length tubular cylinder due to a delta function voltage source of strength, V (volts), (see Figure 1) may be constructed by summing the multiply reflected currents from the ends. The resulting expression is,

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$$I^T(z) = V\{u_\pi^a(|z-z_0|) + C(h-z_0)R_\pi^a u_\pi^a(h-z) + C(h+z_0)R_\pi^a u_\pi^a(h+z)\}; \quad -h<z<h \tag{2}$$

where ,

$$C(z) = \frac{u_\pi^a(2h-z)R_\pi^a u_\pi^a(2h) - u_\pi^a(z)}{1 - [R_\pi^a u_\pi^a(2h)]^2} \tag{3}$$

$$R_\pi^a = \frac{\eta}{2\pi}\,\frac{2Cw + i\pi}{[1 + 0.624(ka) + 0.665(ka)^2]^2}\,; \quad \eta = 120\pi \text{ (ohms)} \tag{4}$$

$$u_\pi^a(z) = \frac{i}{\eta}\, e^{ikz}\, \ell n\,\{1 - \frac{i2\pi}{2Cw+\gamma+i3\pi/2 + \ell n(2kz) + e^{-i2kz}E_1(-i2kz)}\} \tag{5}$$

and

$$Cw = -\ell n(ka) - \gamma, \qquad \gamma = 0.57721 \;\ldots\; . \tag{6}$$

E_1 appearing in (5) is the exponential integral of the first kind. The time convention here has been taken as $e^{-i\omega t}$. $u_\pi^a(z)$ in (5) is an asymptotic (subject to the restriction in (1)) expression for the current on an infinitely-long driven cylindrical antenna. $C(z)$ in (3) with $z = h-z_0$ and $h + z_0$ gives the total incident current upon the end $z = +h$ and $-h$, respectively, due to reflections eminating from the opposite end. Without the terms in the denominator of (4), this expression for the reflection coefficient corresponds exactly to the one derived in [1], which is based upon the Wiener-Hopf procedure for a thin ($ka << 1$) wire. Our modification (via a curve fitting technique) extends the use of this expression to values of ka up to 1. In addition, for the larger values of ka, empirical observations support a significant relaxation of the condition in (1), although for values of kh less than 1, the satisfaction of (1) becomes an increasingly inportant factor to our theory. Current distributions (sufficiently away from the source and ends) obtained from (2)-(6) over a wide range of cylinder radii and lengths can be shown to be consistent with existing theories.

III. Input Conductance of a Cylindrical Antenna

The real part of the current $I^T(z)$ in (2) evaluated at $z = z_0$ gives the input conductance of a dipole antenna of finite length. To achieve greater accuracy in the determination of the conductance, we replace the term $\mathrm{Re}\{u_\pi^a(0)\}$ which corresponds to the input conductance of an infinitely long antenna by

$$G_\infty = \frac{\pi}{\eta}\,\{\frac{-1}{\ell n[\frac{ka}{\sqrt{2}\,[1 + 11.577(ka) - 2.25\,(ka)^2]}] + \gamma} + ka\} \tag{7}$$

which is a modification (again via a curve fitting technique) of Fante's [3] input conductance formula for an infinitely long thin cylinder. Equation (7) now extends the range of applicability of the original expression to values of ka as large as 1. The input conductance of a cylindrical antenna where $\Omega = 2\ell n(2h/a) \doteq 10$ as predicted by (2)-(6) with (7) is shown in Figure 2 as a function of the electrical length, kh. The comparison with King's [2,Sec.

II.30] data for the same antenna, also shown in Figure 2, is quite good over the range $1.0 \leqslant kh \leqslant 7.0$, however, unlike King's theory, our formulation provides conductance values up to $kh = 75$ ($ka = 1.0$). In the range $0.2 \leqslant kh \leqslant 1.0$ we resort to the determination of the input conductance by means of the effective aperture theory given by Chang and Rispin in [4], based upon an expression similar to (2) for a receiving antenna.

As mentioned earlier, the extension of our theory to cylinders as thick as $ka = 1.0$ is accompanied by an observed relaxation of the restriction in (1). To demonstrate this, we offer the conductance data obtained from (2) - (6) with (7) as a function of the electrical half-length, kh, for cylinders having electrical radii of $ka = 0.01$, 0.1, and 0.74 in Figure 3. For comparison, corresponding data from King[2, Sec. II.30] for the electrically thin case, Hartig [2, Sec. II.38] for the moderately thick case, and Chang [6] for the thick antenna case is also shown. Note that even in the electrically thick case, $ka = 0.74$, the behavior of the conductance near the first resonance is fairly well predicted by our simple theory. And for larger values of kh, this simple theory should provide even better results.

Conclusion

A simple theory for the cylindrical transmitting antenna has been given, which is applicable over a wide frequency range. Current distributions and input conductance values may be obtained within the general framework of this theory for antennas varying in length from short to very long and in radius from very thin to quite thick.

References

[1] Shen, L.C., T.T. Wu, and R.W.P. King, "A simple formula of current in transmitting antennas," IEEE Trans. Ant. and Prop., Vol. 16, No. 5, pp. 542-547, Sept. 1968.

[2] King, R.W.P., The Theory of Linear Antennas, Harvard Univ. Press, Cambridge, Mass., 1956.

[3] Harrington, R.F., Field Computation by Moment Methods, Macmillan Co., New York, 1968, Sec. 4-4.

[4] Fante, R.L., "On the admittance of the infinite cylindrical antenna," Rad. Sci., Vol. 1 (New Series), No. 9, pp. 1041-1044, Sept., 1966. And correction to the above, Rad. Sci., Vol. 1 (New Series), No. 10, pp. 1234, Oct., 1966.

[5] D.C.Chang and L.W. Rispin, "The use of effective aperture relations for the calculation of the input conductance of electrically small antenna," submitted for publication to IEEE Trans. Ant. and Prop.

[6] Chang, D.C., "On the electrically thick monopole," IEEE Trans. Ant. and Prop., Vol. 16, No. 1, pp. 58-71, Jan., 1968.

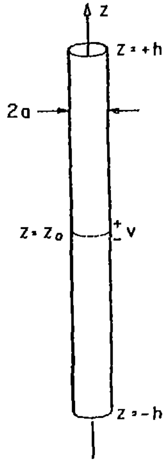

Figure 1. Finite cylindrical transmitting antenna.

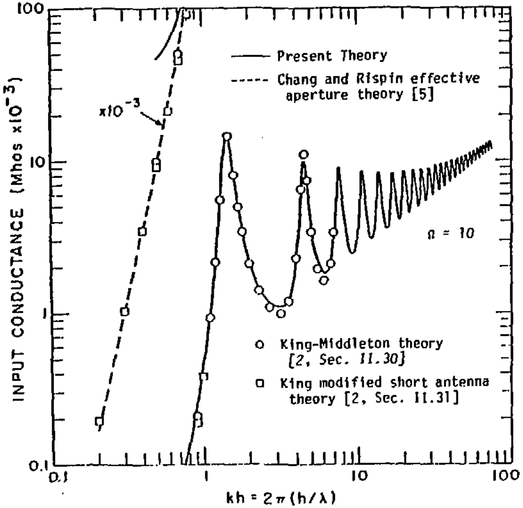

Figure 2. Input conductance for fixed $\Omega = 2\ln(2h/a) = 10$.

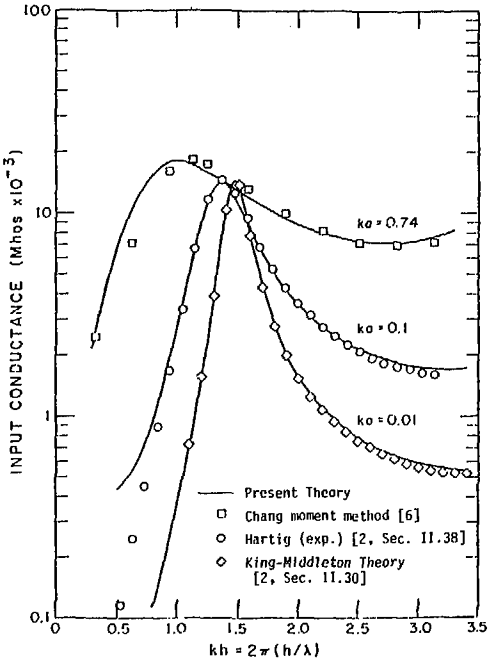

Figure 3. Input conductance for fixed values of ka.